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TECHNICAL NOTES

TRANSMITTANCE OF COLLIMATED RADIATION BY A CYLINDRICAL ABSORBING COLUMN

SANG-CHUN RAH, ADEL F. SAROFIM and JÁNOS M. BEÉR
Massachusetts Institute of Technology, Department of Chemical Engineering, Cambridge, MA 02139, U.S.A.

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NOMENCLATURE

A_p projected area
A_T total surface area
D characteristic dimension (diameter for cylinder or sphere, slab thickness)
I intensity of outgoing beam
I_0 intensity of incoming beam
$I_1(\xi)$ modified Bessel function of order 1
k absorption coefficient
L path length
L_m mean beam length
L_0 mean beam length for optically thin limit
$\mathscr{L}_1(\xi)$ modified Struve function of order 1
$\dot{Q}$ rate of energy absorption
R radius of cylinder or sphere
V volume
x coordinate position (Fig. 1).

Greek symbols
α_g absorptance of gas
ξ characteristic optical thickness, kD
τ overall transmittance.

1. INTRODUCTION

INTEREST in measuring the concentration of absorbents in a cylindrical column using a collimated beam of width greater than that of the column (Fig. 1) provided the need for a mean beam length of the radiation for transmission (in contrast, to the available tabulations on emission [1, 2]).

The mean beam length, L_m, is defined as the length at which the absorptance, α_g, of a gas needs to be evaluated in order to yield the correct rate of energy absorption. For example, for a beam of intensity I_0 and divergence angle $d\Omega$ incident on a volume V with projected area A_p the rate of energy absorption $\dot{Q}$ would equal

$$\dot{Q} = \alpha_g A_p I_0 \, d\Omega, \qquad (1)$$

where α_g was evaluated at L_m, i.e. for a gray gas

$$\alpha_g = 1 - e^{-kL_m},$$

where k is the absorption coefficient of the gas.

Values of the mean beam length in the optically thin limit ($kL \ll 1$), designated by L_0, may be readily obtained. For a collimated beam the rate of energy absorption is equal to the incident flux density $I_0 \, d\Omega$ times kV. Therefore

$$\dot{Q} = kVI_0 \, d\Omega = \alpha_g A_p I_0 \, d\Omega, \qquad (2)$$

or

$$\alpha_g = \frac{kV}{A_p}.$$

Since $\alpha_g \to kL_0$ in the optically thin limit

$$L_0 = \frac{V}{A_p}. \qquad (3)$$

For a randomly oriented objective, the surface of which has no indentation or negative curvature, the projected area A_p is equal to one quarter of the total surface area A_T so that

$$L_0 = \frac{4V}{A_T}. \qquad (4)$$

This is identical to the mean beam length, in the optically thin limit, for gas emission [1, 2]. The results for finite optical thickness are more difficult to evaluate.

2. ANALYSIS AND DISCUSSIONS

Consider collimated radiation passing through an absorbing medium and assume that the attenuation of the intensity of monochromatic radiation along a path follows the Lambert–Bouguer law

$$I(L) = I_0 \, e^{-kL}. \qquad (5)$$

By integrating equation (5) over the volume traversed by the beam we can obtain the overall transmittance as a function of concentration.

The integration of equation (5) for a cylinder of radius R (Fig. 1) is given below

$$\tau_{\text{cylinder}} = \frac{\int_{-R}^{R} I(L) \, dx}{\int_{-R}^{R} I_0 \, dx} = \frac{\int_0^R e^{-kL} \, dx}{R}, \qquad (6)$$

where

$$L = 2\sqrt{(R^2 - x^2)}. \qquad (7)$$

The numerator of equation (6) can be transformed by substituting equation (7) into equation (6) and rearranging as follows:

$$\int_0^R e^{-kL} \, dx = \frac{1}{2} \int_0^D \frac{e^{-kL} L \, dL}{\sqrt{(D^2 - L^2)}}. \qquad (8)$$

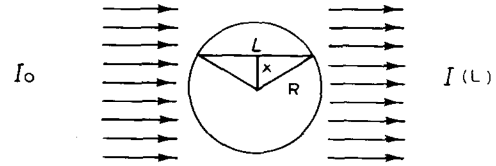

FIG. 1. Collimated radiation passing through a cylinder (top view).

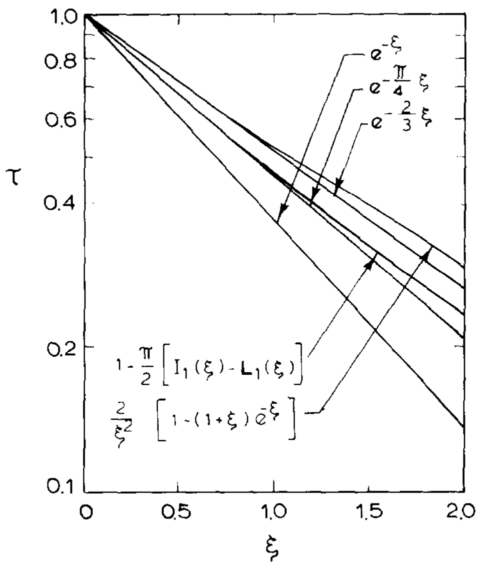

FIG. 2. τ vs ξ and asymptotes.

The integral on the RHS of equation (8) is the Laplace transform of the following function

$$f(L) = \begin{cases} L(D^2-L^2)^{-1/2}; & 0 < L < D, \\ 0; & L < D, \end{cases} \tag{9}$$

and can be evaluated as [3]

$$\int_0^D \frac{e^{-kL} L \, dL}{\sqrt{(D^2-L^2)}} = D - \frac{\pi D}{2}[I_1(\xi) - \mathscr{L}_1(\xi)], \tag{10}$$

where $\xi = kD$, $I_1(\xi)$ is the modified Bessel function of order 1, and $\mathscr{L}_1(\xi)$ is the modified Struve function of order 1. Thus, we obtain

$$\tau_{\text{cylinder}} = 1 - \frac{\pi}{2}[I_1(\xi) - \mathscr{L}_1(\xi)]. \tag{11}$$

For various values of ξ, τ_{cylinder} was calculated by using a table of $I_1(\xi)$ and $\mathscr{L}_1(\xi)$ from ref. [4], and is plotted in Fig. 2.

The transmittance for a plane of thickness D (or a cube of side D) when the radiation is incident normal to a face, and a sphere of diameter D are given for comparison in Fig. 2. It should be noted that the transmittance of radiation for a sphere for collimated and diffuse radiation is identical so that τ for the sphere is given by the well-known [1, 2] relation

$$\tau_{\text{sphere}} = \frac{2}{\xi^2}[1-(1+\xi)e^{-\xi}]. \tag{12}$$

The negative slopes of τ at the origin (see Fig. 2) are equal to L_0/D, or in agreement with the predictions of equation (3), $\pi/4$ for a cylinder, 1 for a slab (or cube), and 2/3 for a sphere.

It can be seen that the error in τ of approximating it by e^{-kL_0} over the range of interest in many practical situations ($kD < 2$) is zero for a slab, and less than 11% for a cylinder and a sphere. The recommendation is therefore made that in the absence of an exact formulation the transmittance of any objective may be adequately approximated for $kD < 2$ by

$$\tau = e^{-kV/A_p}. \tag{13}$$

A lower average error over the interval $0 < kD < 2$ can be obtained by using a smaller mean beam length for the cylinder and sphere, unlike the mean beam length for emission, however, no one correction factor can be defined that applies for all optical thicknesses.

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UNSTEADY THREE-DIMENSIONAL MIXED CONVECTION FLOW WITH TEMPERATURE DEPENDENT VISCOSITY

C. D. SURMA DEVI† and G. NATH‡

Department of Applied Mathematics, Indian Institute of Science, Bangalore 560012, India

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NOMENCLATURE

c	ratio of the velocity gradients at the edge of the boundary layer (b/a) when $t^* = 0$
$C_f, \bar{C}_f$	skin-friction coefficients at the wall in the x- and y-directions, respectively
F, S	dimensionless velocity components in the x- and y-directions, respectively
F'_w, S'_w	skin-friction parameters in the x- and y-directions, respectively
G, G'_w	dimensionless temperature and heat transfer parameter at the wall, respectively
Gr, N	Grashof number and ratio of viscosity, respectively
Nu, Pr	Nusselt and Prandtl numbers, respectively
t^*	dimensionless time

†Present address: Department of Mathematics, Central College, Bangalore University, Bangalore 560001, India.
‡To whom correspondence should be addressed.